Indian Statistical Institute, Bangalore
B. Math II, First Semester, 2024-25
Supplementary Mid-semester Examination, Introduction to Statistical Inference
09.10.24 Maximum Score 65 Duration: 2 Hours

1. (10) Let $X_1, \cdots, X_n$ be independent random variables with the following densities

$$f_{X_k}(x) = \frac{1}{2k\theta} \quad \text{for } -k(\theta-1) < x < k(\theta+1)$$

Show that $(\max \frac{x_k}{k}, \min \frac{x_k}{k})$ is sufficient statistic for θ.

2. (15) Let $X_1, \cdots, X_n$ be i.i.d. observations from $Unif([0,\theta])$.

   (a) Show that the Pareto distribution, given below as π, is a conjugate prior.

$$\pi(\theta|\alpha,\beta) = \frac{\alpha\beta^\alpha}{\theta^{\alpha+1}}, \theta \geq \beta > 0$$

   (b) Find the posterior mean of θ under the Pareto(α,β) prior.

3. (10) Let $\mathbf{X} \sim \text{Bin}(n, \frac{1}{4})$.

   (a) Find the MLE of n when $\mathbf{X} = 5$.

   (b) Find the MoM estimator for n when $\mathbf{X} = 5$.

4. (15) Let $X_1, \cdots, X_n$ be iid Exponential(λ) random variables, with pdf of X_1 being $\lambda e^{-\lambda x}$ for $x > 0$.

   (a) Show that $\bar{X}$ attains the Cramer-Rao lower bound.

   (b) Hence conclude that $\bar{X}$ is UMVUE for $1/\lambda$.

5. (15) Consider the bivariate normal density

$$h(x,y) = \frac{1}{2\pi} \exp\left\{-\frac{1}{2}\left(x^2 + 2y^2 - 2xy - 2x - 6y + 17\right)\right\}$$

Find the mean vector and the covariance matrix.